February 26, 2016	Math 2L03	Dr. Prayat Poudel
Test 2		50 Minutes

Full Name

Solution Key

Student I.D.

THIS EXAMINATION PAPER INCLUDES **8** PAGES AND **5** QUESTIONS. YOU ARE RESPONSIBLE FOR ENSURING THAT YOUR COPY OF THE PAPER IS COMPLETE. BRING ANY DISCREPANCY TO THE ATTENTION OF YOUR INVIGILATOR.

INSTRUCTIONS: No aids except the standard Casio fx991 calculator are permitted.

Problem	Points	Score
1	10	
2	30	
3	20	
4	25	
5	15	
Total:	100	

1. (10 points) Determine if the following statements are True or False. **Explain your answers** (NO CREDITS WILL BE OFFERED IF JUSTIFICATION IS NOT PROVIDED)

   (a) (2 points) If A and B are square matrices of the same order, then $(AB)^T = A^T B^T$.

   FALSE.

   $(AB)^T = B^T A^T$

   (b) (2 points) The sum of two invertible matrices of the same size must be invertible.

   False $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ are both invertible since $\det A = \det B = 1$

   However $A + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ is not invertible since $\det(A+B) = 0$.

   (c) (3 points) The game given by the payoff matrix

$$P = \begin{pmatrix} -2 & -4 & \boxed{9} \\ \boxed{1} & 1 & 0 \\ -1 & -2 & -3 \\ 1 & \boxed{1} & -1 \end{pmatrix}$$

   is strictly determined.

   False Not strictly determined since there are no points which are both row minima & column maxima.

(d) (3 points) If A is square matrix and $A^3 = I$, then A is invertible.

True $A^3 = I$

$\Rightarrow A.A^2 = A^2.A = I$

$\Rightarrow A^{-1} = A^2$

2. (30 points) The question is based on the following data on three bond funds at the following companies:

	Loss
Company U	6 %
Company V	5 %
Company W	4 %

You invested a total of \$9000 in the three bond funds at the beginning of 2014, including an equal amount in Company U and Company V. Your total year-to-date loss amounted to \$480.

(a) (7 points) Find the matrix equation $AX = B$ which models to solve the problem.

$x \equiv$ company U, $y \equiv$ comp. V, $z \equiv$ comp. W

$$x + y + z = 9000$$
$$x - y = 0$$
$$0.06x + 0.05y + 0.04z = 480$$

$$\underbrace{\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 0 \\ 0.06 & 0.05 & 0.04 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{X} = \underbrace{\begin{bmatrix} 9000 \\ 0 \\ 480 \end{bmatrix}}_{B}$$

(b) (18 points) Find the inverse of A which you found above.

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 0.06 & 0.05 & 0.04 & 0 & 0 & 1 \end{array}\right] 100R_3 \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 6 & 5 & 4 & 0 & 0 & 100 \end{array}\right] \begin{array}{l} \\ R_2 - R_1 \\ R_3 - 6R_1 \end{array} \longrightarrow$$

$$\longrightarrow \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & -2 & -1 & -1 & 1 & 0 \\ 0 & -1 & -2 & -6 & 0 & 100 \end{array}\right] \begin{array}{l} 2R_1 + R_2 \\ \\ 2R_3 - R_2 \end{array} \longrightarrow \left[\begin{array}{ccc|ccc} 2 & 0 & 1 & 1 & 1 & 0 \\ 0 & -2 & -1 & -1 & 1 & 0 \\ 0 & 0 & -3 & -11 & -1 & 200 \end{array}\right] \begin{array}{l} 3R_1 + R_3 \\ 3R_2 - R_3 \\ \\ \end{array}$$

$$\longrightarrow \left[\begin{array}{ccc|ccc} 6 & 0 & 0 & -8 & 2 & 200 \\ 0 & -6 & 0 & 8 & 4 & -200 \\ 0 & 0 & -3 & -11 & -1 & 200 \end{array}\right] \begin{array}{l} \frac{1}{6}R_1 \\ -\frac{1}{6}R_2 \\ -\frac{1}{3}R_3 \end{array} \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -4/3 & 1/3 & \frac{100}{3} \\ 0 & 1 & 0 & -4/3 & -2/3 & \frac{100}{3} \\ 0 & 0 & 1 & 11/3 & 1/3 & \frac{-200}{3} \end{array}\right]$$

So, $$A^{-1} = \begin{bmatrix} -\frac{4}{3} & \frac{1}{3} & \frac{100}{3} \\ -\frac{4}{3} & -\frac{2}{3} & \frac{100}{3} \\ \frac{11}{3} & \frac{1}{3} & -\frac{200}{3} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} -4 & 1 & 100 \\ -4 & -2 & 100 \\ 11 & 1 & -200 \end{bmatrix}$$

(c) (5 points) Find how much you invested in Company V.

$$X = A^{-1}B = \frac{1}{3}\begin{bmatrix} -4 & 1 & 100 \\ -4 & -2 & 100 \\ 11 & 1 & -200 \end{bmatrix}\begin{bmatrix} 9000 \\ 0 \\ 480 \end{bmatrix} = \begin{bmatrix} 4000 \\ 4000 \\ 1000 \end{bmatrix}$$

Invested \$ 4000 in company V.

3. (20 points) Consider the game with payoff matrix

$$P = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix}$$

(a) (8 points) Find the optimal row strategy (i.e. optimal strategy for the row player).

$e = RPC$, $R = [x \;\; 1-x]$

$$e_1 = [x \;\; 1-x]\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = [x \;\; 1-x]\begin{bmatrix} 2 \\ -1 \end{bmatrix} = 2x + x - 1 = 3x - 1$$

$$e_2 = [x \;\; 1-x]\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = [x \;\; 1-x]\begin{bmatrix} 1 \\ 3 \end{bmatrix} = x + 3 - 3x = 3 - 2x$$

$$3x - 1 = 3 - 2x \Rightarrow x = \frac{4}{5} \;,\; R = \left[\frac{4}{5} , \frac{1}{5}\right]$$

(b) (8 points) Find the optimal column strategy (i.e. optimal strategy for the column player).

$C = [x \;\; 1-x]^T$

$$e_1 = [1 \;\; 0]\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ 1-x \end{bmatrix} = [2 \;\; 1]\begin{bmatrix} x \\ 1-x \end{bmatrix} = 2x + 1 - x = x + 1$$

$$e_2 = [0 \;\; 1]\begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} x \\ 1-x \end{bmatrix} = [-1 \;\; 3]\begin{bmatrix} x \\ 1-x \end{bmatrix} = -x + 3 - 3x = 3 - 4x$$

$$3 - 4x = x + 1 \Rightarrow 2 = 5x \Rightarrow x = \frac{2}{5} \;,\; C = \begin{bmatrix} 2/5 \\ 3/5 \end{bmatrix}$$

(c) (4 points) Find the expected value of the game in the event that each player uses his/her optimal strategy.

$$e = \begin{bmatrix} \frac{4}{5} & \frac{1}{5} \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 2/5 \\ 3/5 \end{bmatrix} = \frac{35}{25} = \frac{7}{5}$$

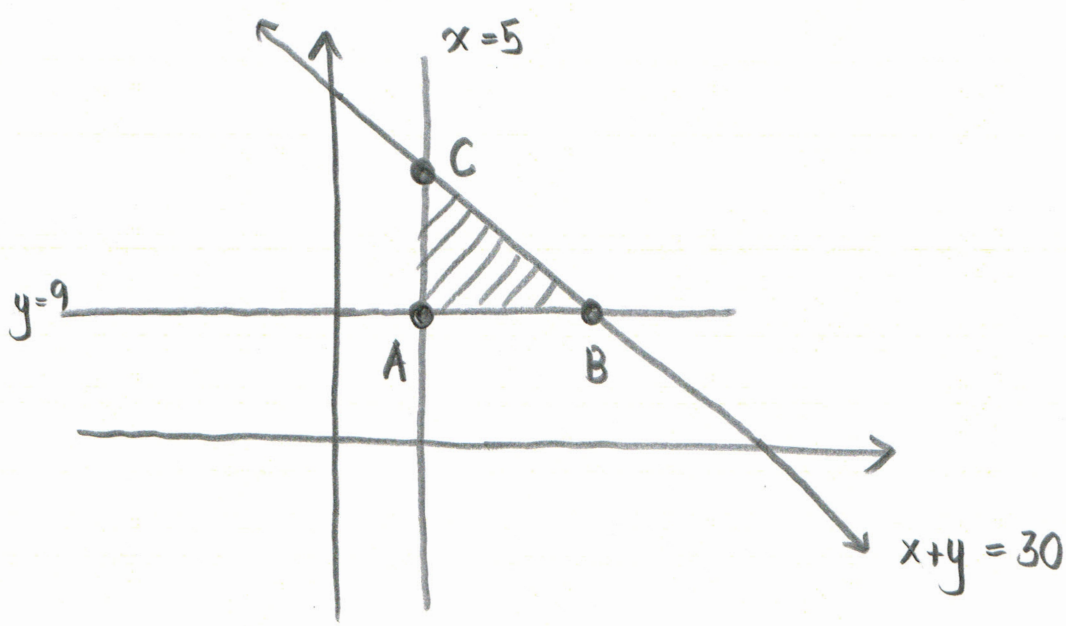
4. (25 points) Fly-Away Airlines sells business class and economy class seats for its charter flights. To charter a plane at least 5 business class tickets must be sold and at least 9 economy class tickets must be sold. The plane does not hold more than 30 passengers.

(a) (12 points) Draw the feasible region that shows the number of business class and economy class seats.

x = # business class

y = # econ. class

$x \geq 5$, $y \geq 9$, $x + y \leq 30$



(b) (7 points) Find the corner points of the feasible region.

A : (5,9)

B : (5,25)

C : (21,9)

(c) (6 points) Fly-Away makes \$40 profit for each business class ticket sold and \$45 profit for each economy class ticket sold. In order for Fly-Away Airlines to maximize its profits, how many economy class seats should they sell?

$P(x) = 40x + 45y$ (Max occurs at corner points)

A	(5,9)	P = 405
B	(5,25)	P = 1325 max
C	(21,9)	P = 1245

So they should sell 25 economy tickets.

5. (15 points) *There is no partial credit. Your mark will be based solely on what is written in the box provided*

(a) (10 points) We want to solve the following Standard Maximization Problem:

$$\underline{\text{Maximize:}} \quad p = x - y + z$$

$$\begin{aligned} \underline{\text{Subject to:}} \quad & x + y + z \leq 3 \\ & 3x - 2y + 6z \leq 4 \\ & x \geq 0;\ y \geq 0;\ z \geq 0 \end{aligned}$$

Find the initial tableau.

	x	y	z	s	t	p	
s	1	1	1	1	0	0	3
t	3	-2	6	0	1	0	4
p	-1	1	-1	0	0	1	0

(b) (5 points) Reduce the payoff matrices by dominance.

$$P = \begin{pmatrix} 1 & -1 & -5 \\ 4 & 0 & 2 \\ 3 & -3 & 10 \\ 3 & -5 & -4 \end{pmatrix}$$

(0)